NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES
SCHOOL OF NATURAL AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSAM	**LEVEL:** 7
COURSE CODE: MMO701S	**COURSE NAME:** MATHEMATICAL MODELLING 1
SESSION: JUNE 2023	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 130 (to be converted to 100%)

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINERS	PROF. S. A. REJU
MODERATOR:	PROF. O. D. MAKINDE

INSTRUCTIONS

1. Attempt ALL the questions.
2. All written work must be done in blue or black ink and sketches must be done in pencils.
3. Use of COMMA is not allowed as a DECIMAL POINT.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (including this front page)

QUESTION 1 [30 MARKS]

(a) Formulate two different models that estimate the proportionality of the elongation to the mass from the following experimental data of a mass-spring system:

Mass (m)	40	87	130	182	220	275	325
Elongation (x)	1.40	1.75	2.25	3.15	4.30	5.35	6.20

Clearly show how your proportionality constant is obtained for each model (correct to 4 decimal places). **[20]**

(b) (i) Characterise the method of Conjecture in Mathematical modelling.
(ii) Show that the solution of the dynamical system

$$a_{n+1} = ra_n + b,\ r \neq 1 \tag{1.1}$$

Is given by

$$a_k = r^k c + \frac{b}{1-r} \tag{1.2}$$

for some c (which depends on the initial condition). **[10]**

QUESTION 2 [30 MARKS]

(a) Consider the following table showing the experimental data of the growth of a micro-organism

n	0	1	2	3	4	5	6
y_n	12.7	18.3	29.2	46.5	71.1	120.1	175.2
Δy_n	8.9	11.7	16.4	23.9	52.2	55.5	86.6

where n is the time in days and y_n is the observed organism biomass.

(i) Formulate a linear model for the above organism and show that the model predicts an increasing population without limit.
(ii) Assume that contrary to your model prediction in (i), there is a maximum population of 325. Hence formulate a nonlinear dynamical system model for the organism growth using your constant obtained from an appropriate ratio similar to the example given in class, for $n = 3$ in the above data.

[15]

(b) Suppose a certain drug is effective in treating a disease if the concentration remains above 120 mg/L. The initial concentration is 645 mg/L. It is known from laboratory experiments that the drug decays at the rate of 25% of the amount present each hour.
(i) Formulate a model representing the concentration at each hour.
(ii) Build a table of values (answer correct to 2 decimal places) and determine when the concentration reaches 120 mg/L.

[15]

QUESTION 3 [40 MARKS]

(a) Consider the following data for bluefish harvesting (in lb) for the years shown.

Year	Blue Fish
1940	15,000
1945	150,000
1950	250,000
1955	275,000
1960	270,000
1965	280,000

Using 1940 as the base year represented by $x = 0$ for numerical convenience, construct a **SINGLE TERM MODEL** for the fish harvesting and hence predict the weight y (lb) of the fish harvested in 2025. HINT: Employ the least squares fit of the model form $\log y = mx + b$ for your procedure, where log is to base 10. **[15]**

(b) Consider the following table of data:

X	1.2	2.5	3.6	4.5	6.5	7.2
y	3.5	3.2	5.7	6.2	4.6	7.7

(i) Estimate the coefficients of the straight line $y = ax + b$ such that the sum of the squared deviations of the data points and the line is minimised.

(ii) State the general normal equations arising from the use of least squares criterion for your answer in (i) and hence obtain the normal equations from your data.

(iii) State the MATLAB commands to obtain the parameters a and b.

(iv) If the largest absolute deviations for the Chebyshev's criterion and that of the Least Squares criterion are given respectively by c_{max} and d_{max}, define them and then compute their values including their least bound D to express their relationship for the above data and the model line. **[25]**

QUESTION 4 [30 MARKS]

(a) A sewage treatment plant processes raw sewage to produce usable fertilizer and clean water by removing all other contaminants. The process is such that each other 13.5% of remaining contaminants in a processing tank are removed.

i. What percentage of the sewage would remain after half a day?

ii. How long would it take to lower the amount of sewage by half?

iii. How long until the level of sewage is down to 12.5% of the original level?

[15]

(b) Construct natural cubic splines that pass through the following data points.

x_i	0	1	2
y_i	0	5	8

[15]

END OF QUESTION PAPER TOTAL MARKS = 130